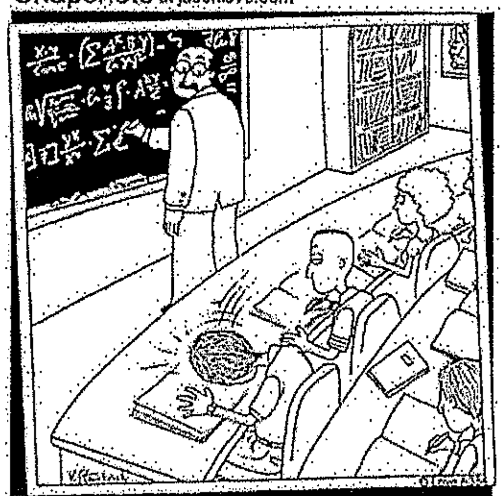
Name: ______________________________

Glenbard East High School Calculus Summer Review Packet

2016 - 2017

Snapshots at jasonlove.com



Professor Herman stopped when he heard that unmistakable thud -- another brain had imploded.

1. This packet is to be handed in to your Calculus teacher on Friday, (1st full week)
2. All work must be shown in the packet OR on a separate sheet of paper attached to the packet.
3. Appropriate completion of this packet will be worth a 50 point Essential Practice for Learning assignment. *If this packet is completed and turned in on the first day of school then the student will be allowed to drop their lowest quiz grade in the first quarter.
4. Students are expected to e-mail Mr. Rogowski (thomas.rogowski@glenbard.org) if they have any questions or research how to do a particular topic on their own by using You Tube, Khan Academy, or Hippocampus.
5. Answers to odd-numbered problems have been provided.
6. If a student has any other questions or concerns about the packet please see Mr. Rogowski or Mr. Sabol before the end of this school year.
7. Please show all work.

Functions

> **To evaluate a function for a given value, simply plug the value into the function for x.**
>
> **Recall:** $(f \circ g)(x) = f(g(x))$ OR $f[g(x)]$ read "*f* of *g* of *x*" means to plug the inside function (in this case g(*x*) in for *x* in the outside function (in this case, *f*(*x*)).
>
> **Example:** Given $f(x) = 2x^2 + 1$ and $g(x) = x - 4$ find $f(g(x))$.
>
> $$\begin{aligned} f(g(x)) &= f(x-4) \\ &= 2(x-4)^2 + 1 \\ &= 2(x^2 - 8x + 16) + 1 \\ &= 2x^2 - 16x + 32 + 1 \\ f(g(x)) &= 2x^2 - 16x + 33 \end{aligned}$$

Let $f(x) = 2x + 1$ and $g(x) = 2x^2 - 1$. Find each of the following.

6) $f(2) =$

7) $g(-3) =$

8) $f(t+1) =$

9) $f[g(-2)] =$

10) $g[f(m+2)] =$

11) $\dfrac{f(x+h) - f(x)}{h} =$

Let $f(x) = \sin x$. Find each exactly.

12) $f\left(\dfrac{\pi}{2}\right) =$

13) $f\left(\dfrac{2\pi}{3}\right) =$

Let $f(x) = x^2$, $g(x) = 2x + 5$, and $h(x) = x^2 - 1$. Find each of the following.

14) $h[f(-2)] =$

15) $f[g(x-1)] =$

16) $g[h(x^3)] =$

Use substitution or elimination method to solve the system of equations.
Example:

$$x^2+y-16x+39=0$$
$$x^2-y^2-9=0$$

Elimination Method

$2x^2-16x+30=0$

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$x=3$ and $x=5$

Plug $x=3$ and $x=5$ into one original

$3^2-y^2-9=0$	$5^2-y^2-9=0$
$-y^2=0$	$16=y^2$
$y=0$	$y=\pm4$

Points of Intersection (5, 4), (5, -4) and (3, 0)

Substitution Method

Solve one equation for one variable.

$y^2=-x^2+16x-39$ (1st equation solved for y)

$x^2-(-x^2+16x-39)-9=0$ Plug what y^2 is equal to into second equation.

$2x^2-16x+30=0$ *(The rest is the same as previous example)*

$x^2-8x+15=0$

$(x-3)(x-5)=0$

$x=3$ or $x=5$

Find the point(s) of intersection of the graphs for the given equations.

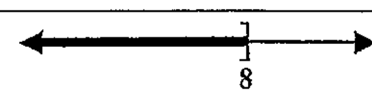
23) $x+y=8$
$4x-y=7$

24) $x^2+y=6$
$x+y=4$

25) $x^2-4y^2-20x-64y-172=0$
$16x^2+4y^2-320x+64y+1600=0$

Interval Notation

26) Complete the table with the appropriate notation or graph.

Solution	Interval Notation	Graph
$-2<x\leq4$		
	$[-1,7)$	
		

Also, recall that to PROVE one function is an inverse of another function, you need to show that:
$f(g(x)) = g(f(x)) = x$

Example:

If: $f(x) = \frac{x-9}{4}$ and $g(x) = 4x+9$ **show** $f(x)$ **and** $g(x)$ **are inverses of each other.**

$$f(g(x)) = 4\left(\frac{x-9}{4}\right)+9 \qquad g(f(x)) = \frac{(4x+9)-9}{4}$$

Prove f **and** g **are inverses of each other.**

36) $f(x) = \frac{x^3}{2}$ $\quad g(x) = \sqrt[3]{2x}$

37) $f(x) = 9 - x^2, x \geq 0$ $\quad g(x) = \sqrt{9-x}$

Radian and Degree Measure

Use $\dfrac{180}{\pi \; radians}$ to get rid of radians and convert to degrees.

Use $\dfrac{\pi \; radians}{180}$ to get rid of degrees and convert to radians.

46) **Convert to degrees:** a) $\dfrac{5\pi}{6}$ b) $\dfrac{4\pi}{5}$ c) 2.63 radians

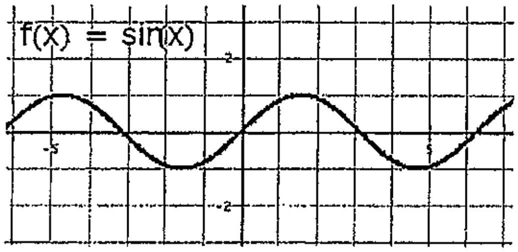
47) **Convert to radians:** a) 45 b) -17 c) 237

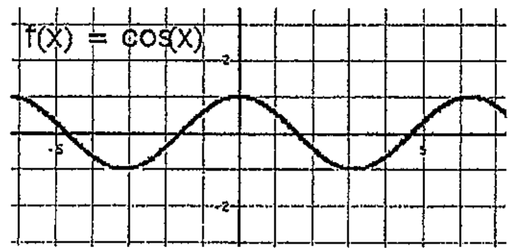
Angles in Standard Position

48) Sketch the angle in standard position.

a) $\dfrac{11\pi}{6}$ b) 230 c) $-\dfrac{5\pi}{3}$ d) 1.8 radians

Graphing Trig Functions





$y=\sin x$ and $y=\cos x$ have a period of 2π and an amplitude of 1. Use the parent graphs above to help you sketch a graph of the function below. For $f(x)=A\sin(Bx+C)+K$, A = amplitude, $\frac{2\pi}{B}$ =period, $\frac{C}{B}$ =phase shift (positive $\frac{C}{B}$ shift left, negative $\frac{C}{B}$ shift right) and K = vertical shift

Graph two complete periods of the function.

51) $f(x)=5\sin x$

52) $f(x)=\sin 2x$

53) $f(x)=-\cos\left(x-\frac{\pi}{4}\right)$

54) $f(x)=\cos x-3$

Trigonometric Equations

Solve each of the equations for $0\le x<2\pi$. Isolate the variable, sketch a reference triangle, find all the solutions within the given domain, $0\le x<2\pi$. Remember to double the domain when solving for a double angle. Use trig identities, if needed , to rewrite the trig functions. (See formula sheet at the end of the packet.)

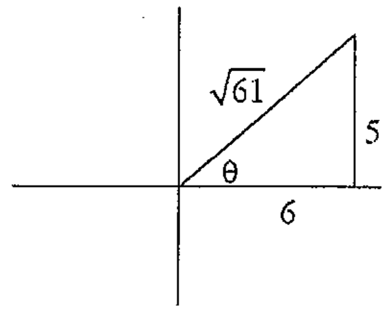
55) $\sin x=-\frac{1}{2}$

56) $2\cos x=\sqrt{3}$

Example: Find the value without a calculator.

$$\cos\left(\arctan\frac{5}{6}\right)$$

Draw the reference triangle in the correct quadrant first.
Find the missing side using Pythagorean Theorem.
Find the ratio of the cosine of the reference triangle.



$$\cos\theta = \frac{6}{\sqrt{61}}$$

For each of the following give the value without a calculator.

63) $\tan\left(\arccos\frac{2}{3}\right)$

64) $\sec\left(\sin^{-1}\frac{12}{13}\right)$

65) $\sin\left(\arctan\frac{12}{5}\right)$

66) $\sin\left(\sin^{-1}\frac{7}{8}\right)$

Limits

Finding limits numerically.

Complete the table and use the result to estimate the limit.

71) $\lim_{x \to 4} \frac{x-4}{x^2-3x-4} =$

x	3.9	3.99	3.999	4.001	4.01	4.1
$f(x)$						

72) $\lim_{x \to -5} \frac{\sqrt{4-x}-3}{x+5} =$

x	-5.1	-5.01	-5.001	-4.999	-4.99	-4.9
$f(x)$						

Find the limits graphically.

Find each limit graphically. Use your calculator to assist in graphing.

73) $\lim_{x \to 0} \cos x$

74) $\lim_{x \to 5} \frac{2}{x-5}$

75) $\lim_{x \to 1} f(x)$

$$f(x) = \begin{cases} x^2+3, & x \neq 1 \\ 2, & x = 1 \end{cases}$$

Evaluating Limits Analytically

Solve by direct substitution whenever possible. If needed, rearrange the expression so that you can do direct substitution.

76) $\lim_{x \to 2} (4x^2+3)$

77) $\lim_{x \to 1} \frac{x^2+x+2}{x+1}$

78) $\lim_{x \to 0} \sqrt{x^2+4}$

79) $\lim_{x \to \pi} \cos x$

Vertical Asymptotes
Determine the vertical asymptotes for the function. Set the denominator equal to zero to find the x-value for which the function is undefined. That will be the vertical asymptote.

89) $f(x)=\frac{1}{x^2}$

90) $f(x)=\frac{x^2}{x^2-4}$

91) $f(x)=\frac{2+x}{x^2(1-x)}$

Horizontal Asymptotes
Determine the horizontal asymptotes using the three cases below.

Case I: Degree of the numerator is less than the degree of the denominator. The asymptote is $y=0$.

Case II: Degree of the numerator is the same as the degree of the denominator. The asymptote is the ratio of the lead coefficients.

Case III: Degree of the numerator is greater than the degree of the denominator. There is no horizontal asymptote. The function increases without bound. (If the degree of the numerator is exactly 1 more than the degree of the denominator, then there exists a slant asymptote, which is determined by long division.)

Determine all Horizontal Asymptotes.

92) $f(x)=\frac{x^2-2x+1}{x^3+x-7}$

93) $f(x)=\frac{5x^3-2x^2+8}{4x-3x^3+5}$

94) $f(x)=\frac{4x^5}{x^2-7}$

Formula Sheet

Reciprocal Identities

$\csc x = \frac{1}{\sin x}$

$\sec x = \frac{1}{\cos x}$

$\cot x = \frac{1}{\tan x}$

Quotient Identities

$\tan x = \frac{\sin x}{\cos x}$

$\cot x = \frac{\cos x}{\sin x}$

Pythagorean Identities

$\sin^2 x + \cos^2 x = 1$

$\tan^2 x + 1 = \sec^2 x$

$1 + \cot^2 x = \csc^2 x$

Double Angle Identities:

$\sin 2x = 2\sin x \cos x$

$\tan 2x = \frac{2\tan x}{1 - \tan^2 x}$

$$\begin{aligned} \cos 2x &= \cos^2 x - \sin^2 x \\ &= 1 - 2\sin^2 x \\ &= 2\cos^2 x - 1 \end{aligned}$$

Logarithms

$y = \log_a x$ is equivalent to $x = a^y$

Product Property

$\log_b mn = \log_b m + \log_b n$

Quotient Property

$\log_b \frac{m}{n} = \log_b m - \log_b n$

Power Property

$\log_b m^p = p \log_b m$

Property of Equality

If $\log_b m = \log_b n$, then $m = n$

Change of Base Formula

$\log_a n = \frac{\log_b n}{\log_b a}$

Derivative of a Function

Slope of a tangent line to a curve or the derivative: $\lim\limits_{h \to \infty} \frac{f(x+h) - f(x)}{h}$

Slope-Intercept Form

$y = mx + b$

Point-Slope Form

$y - y_1 = m(x - x_1)$

Standard Form

$Ax + By + C = 0$

59. $x=\left\{\frac{2\pi}{3},\frac{5\pi}{3},\frac{5\pi}{6},\frac{11\pi}{6}\right\}$ 61. $x=\left\{\frac{\pi}{6},\frac{5\pi}{6},\frac{7\pi}{6},\frac{11\pi}{6}\right\}$ 63. $\frac{\sqrt{5}}{2}$ 65. $\frac{12}{13}$

67. circle with center (0 , 0) and radius 4 69. Ellipse with center (0, 0) and vertical major axis length 6 and minor horizontal axis of 2 71. $\frac{1}{5}$ 73. 1

75. 4 77. 2 79. -1 81. -5 83. $-\frac{1}{6}$

85. $\frac{1}{10}$ 87. 1 89. VA $x=0$ 91. VA $x=0$, $x=1$ 93. None

95. 4 97. ∞ *or DNE* 99. DNE